1.3 Vertical Motion

Catapulting a diver high into the air requires a force. How large a force? How hard must the board push up on the diver to overcome her weight and accelerate her upward? After the diver leaves the board, how long will it take before her ascent stops and she turns and plunges toward the water? In this section, you will investigate the dynamics of diving and other motions involving rising and falling or straight-line motion in a vertical plane.

Figure 1.8 After the diver leaves the diving board and before she hits the water, the most important force acting on her is the gravitational force directed downward. Gravity affects all forms of vertical motion.

SECTION EXPECTATIONS

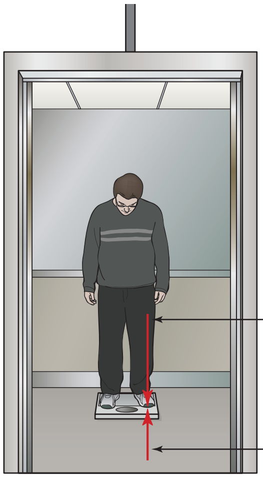
- Analyze the motion of objects in vertical planes.
- Explain linear vertical motion in terms of forces.
- Solve problems and predict the motion of objects in vertical planes.

KEY TERMS

- apparent weight
- tension
- counterweight
- free fall
- air resistance
- terminal velocity

Weight versus Apparent Weight

One of the most common examples of linear vertical motion is riding in an elevator. You experience some strange sensations when the elevator begins to rise or descend or when it slows and comes to a stop. For example, if you get on at the first floor and start to go up, you feel heavier for a moment. In fact, if you are carrying a book bag or a suitcase, it feels heavier, too. When the elevator slows and eventually stops, you and anything you are carrying feels lighter. When the elevator is moving at a constant velocity, however, you feel normal. Are these just sensations that living organisms feel or, if you were standing on a scale in the elevator, would the scale indicate that you *were* heavier? You can answer that question by applying Newton's laws of motion to a person riding in an elevator.



Imagine that you are standing on a scale in an elevator, as shown in Figure 1.9. When the elevator is standing still, the reading on the scale is your weight. Recall that your weight is the force of gravity acting on your mass. Your weight can be calculated by using the equation $F_g = mg$, where g is the acceleration due to gravity. Vector notations are sometimes omitted because the force due to gravity is always directed toward the centre of Earth. Find out what happens to the reading on the scale by studying the following sample problem.

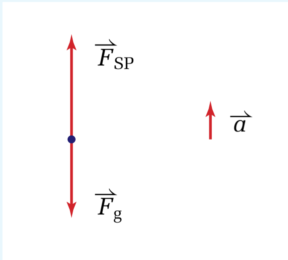
Figure 1.9 When you are standing on a scale, you exert a force on the scale. According to Newton's third law, the scale must exert an equal and opposite force on you. Therefore, the reading on the scale is equal to the force that you exert on it.

SAMPLE PROBLEM

Apparent Weight

A 55 kg person is standing on a scale in an elevator. If the scale is calibrated in newtons, what is the reading on the scale when the elevator is not moving? If the elevator begins to accelerate upward at 0.75 m/s^2, what will be the reading on the scale?

Conceptualize the Problem



- Start framing the problem by drawing a *free-body diagram* of the person on the scale. A free-body diagram includes all of the *forces acting on the person*.
- The *forces* acting on the person are *gravity* ($\vec{F}_g$) and the *normal force* of the scale.
- According to *Newton's third law*, when the person exerts a *force* ($\vec{F}_{PS}$) on the scale, it exerts an *equal and opposite force* ($\vec{F}_{SP}$) on the person. Therefore, the reading on the scale is the same as the force that the person exerts on the scale.
- When the elevator is *standing still*, the person's *acceleration is zero*.
- When the elevator begins to *rise*, the person is *accelerating* at the same rate as the elevator.
- Since the motion is in one dimension, use only positive and negative signs to indicate direction. Let "*up*" *be positive* and "*down*" *be negative*.
- Apply *Newton's second law* to find the magnitude of $\vec{F}_{SP}$.
- By *Newton's third law*, the magnitudes of $\vec{F}_{PS}$ and $\vec{F}_{SP}$ are equal to each other, and therefore to the reading on the scale.

Identify the Goal

The reading on the scale, $|\vec{F}_{SP}|$, when the elevator is standing still and when it is accelerating upward

Identify the Variables

Known	Implied	Unknown
$m = 55$ kg	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	$\vec{F}_{PS}$ $\quad \vec{F}_{SP}$
$\vec{a} = +0.75\ \frac{\text{m}}{\text{s}^2}$		$\vec{F}_g$

Develop a Strategy

Apply Newton's second law and solve for the force that the scale exerts on the person. The force in Newton's second law is the vector sum of all of the forces acting *on* the person.

In the first part of the problem, the acceleration is zero.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$
$$\vec{F}_{SP} = (55\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + 0$$
$$\vec{F}_{SP} = 539.55\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$
$$\vec{F}_{SP} \cong 5.4 \times 10^2\text{ N}$$

When the elevator is not moving, the reading on the scale is 5.4×10^2 N, which is the person's weight.

Apply Newton's second law to the case in which the elevator is accelerating upward. The acceleration is positive.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_g + \vec{F}_{SP} = m\vec{a}$$
$$\vec{F}_{SP} = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_{SP} = -(-mg) + m\vec{a}$$
$$\vec{F}_{SP} = (55\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (55\text{ kg})\left(+0.75\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_{SP} = 580.8\text{ N}$$
$$\vec{F}_{SP} \cong 5.8 \times 10^2\text{ N[up]}$$

When the elevator is accelerating upward, the reading on the scale is 5.8×10^2 N.

Validate the Solution

When an elevator first starts moving upward, it must exert a force that is greater than the person's weight so that, as well as supporting the person, an additional force causes the person to accelerate. The reading on the scale should reflect this larger force. It does. The acceleration of the elevator was small, so you would expect that the increase in the reading on the scale would not increase by a large amount. It increased by only about 7%.

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PRACTICE PROBLEMS

11. A 64 kg person is standing on a scale in an elevator. The elevator is rising at a constant velocity but then begins to slow, with an acceleration of $0.59\ m/s^2$. What is the sign of the acceleration? What is the reading on the scale while the elevator is accelerating?

12. A 75 kg man is standing on a scale in an elevator when the elevator begins to descend with an acceleration of $0.66\ m/s^2$. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?

13. A 549 N woman is standing on a scale in an elevator that is going down at a constant velocity. Then, the elevator begins to slow and eventually comes to a stop. The magnitude of the acceleration is $0.73\ m/s^2$. What is the direction of the acceleration? What is the reading on the scale while the elevator is accelerating?

As you saw in the problems, when you are standing on a scale in an elevator that is accelerating, the reading on the scale is not the same as your true weight. This reading is called your **apparent weight**.

When the direction of the acceleration of the elevator is positive — it starts to ascend or stops while descending — your apparent weight is greater than your true weight. You feel heavier because the floor of the elevator is pushing on you with a greater force than it is when the elevator is stationary or moving with a constant velocity.

When the direction of the acceleration is negative — when the elevator is rising and slows to a stop or begins to descend — your apparent weight is smaller than your true weight. The floor of the elevator is exerting a force on you that is smaller than your weight, so you feel lighter.

Figure 1.10 Mobile construction cranes can withstand the tension necessary to lift loads of up to 1000 t.

Tension in Ropes and Cables

While an elevator is supporting or lifting you, what is supporting the elevator? The simple answer is cables — exceedingly strong steel cables. Construction cranes such the one in Figure 1.10 also use steel cables to lift building materials to the top of skyscrapers under construction. When a crane exerts a force on one end of a cable, each particle in the cable exerts an equal force on the next particle in the cable, creating tension throughout the cable. The cable then exerts a force on its load. **Tension** is the magnitude of the force exerted on and by a cable, rope, or string. How do engineers determine the amount of tension that these cables must be able to withstand? They apply Newton's laws of motion.

To avoid using complex mathematical analyses, you can make several assumptions about cables and ropes that support loads. Your results will be quite close to the values calculated by computers that are programmed to take into account all of the non-ideal conditions. The simplifying assumptions are as follows.

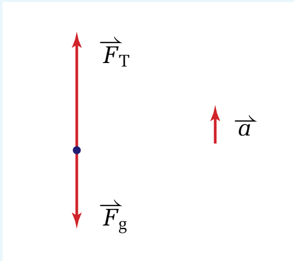
- The mass of the rope or cable is so much smaller than the mass of the load that it does not significantly affect the motion or forces involved.
- The tension is the same at every point in the rope or cable.
- If a rope or cable passes over a pulley, the direction of the tension forces changes, but the magnitude remains the same. This statement is the same as saying that the pulley is frictionless and its mass is negligible.

SAMPLE PROBLEM

Tension in a Cable

An elevator filled with people has a total mass of 2245 kg. As the elevator begins to rise, the acceleration is 0.55 m/s^2. What is the tension in the cable that is lifting the elevator?

Conceptualize the Problem



- To begin framing the problem, draw a free-body diagram.
- The *tension* in the cable has the *same magnitude* as the *force* it exerts on the elevator.
- *Two forces* are acting on the elevator: the *cable* ($\vec{F}_T$) and gravity ($\vec{F}_g$).
- The elevator is *rising* and speeding up, so the *acceleration* is *upward*.
- *Newton's second law* applies to the problem.
- The motion is in *one dimension*, so let positive and negative *signs* indicate *direction*. Let "*up*" *be positive* and "*down*" *be negative*.

Identify the Goal

The tension, F_T, in the rope

Identify the Variables

Known	Implied	Unknown
$m = 2245$ kg	$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$	$\vec{F}_T$
$\vec{a} = 0.55\ \dfrac{\text{m}}{\text{s}^2}$[up]		$\vec{F}_g$

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Develop a Strategy

Apply Newton's second law and insert all of the forces acting on the elevator. Then solve for the tension.

$$\vec{F} = m\vec{a}$$
$$\vec{F}_T + \vec{F}_g = m\vec{a}$$
$$\vec{F}_T = -\vec{F}_g + m\vec{a}$$
$$\vec{F}_T = -(-mg) + m\vec{a}$$

Substitute values and solve.

$$\vec{F} = (2245\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) + (2245\ \text{kg})\left(0.55\ \frac{\text{m}}{\text{s}^2}\right)$$
$$\vec{F}_T = 23\ 258.2\ \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$
$$\vec{F}_T \cong 2.3 \times 10^4\ \text{N[up]}$$

The magnitude of the tension in the cable is 2.3×10^4 N[up].

Validate the Solution

The weight of the elevator is $(2245\ \text{kg})(9.81\ \text{m/s}^2) \cong 2.2 \times 10^4$ N. The tension in the cable must support the weight of the elevator and exert an additional force to accelerate the elevator. Therefore, you would expect the tension to be a little larger than the weight of the elevator, which it is.

PRACTICE PROBLEMS

14. A 32 kg child is practising climbing skills on a climbing wall, while being belayed (secured at the end of a rope) by a parent. The child loses her grip and dangles from the belay rope. When the parent starts lowering the child, the tension in the rope is 253 N. Find the acceleration of the child when she is first being lowered.

15. A 92 kg mountain climber rappels down a rope, applying friction with a figure eight (a piece of climbing equipment) to reduce his downward acceleration. The rope, which is damaged, can withstand a tension of only 675 N. Can the climber limit his descent to a constant speed without breaking the rope? If not, to what value can he limit his downward acceleration?

16. A 10.0 kg mass is hooked on a spring scale fastened to a hoist rope. As the hoist starts moving the mass, the scale momentarily reads 87 N. Find

(a) the direction of motion

(b) the acceleration of the mass

(c) the tension in the hoist rope

17. Pulling on the strap of a 15 kg backpack, a student accelerates it upward at $1.3\ \text{m/s}^2$. How hard is the student pulling on the strap?

18. A 485 kg elevator is rated to hold 15 people of average mass (75 kg). The elevator cable can withstand a maximum tension of 3.74×10^4 N, which is twice the maximum force that the load will create (a 200% safety factor). What is the greatest acceleration that the elevator can have with the maximum load?

Connected Objects

Imagine how much energy it would require to lift an elevator carrying 20 people to the main deck of the CN Tower in Toronto, 346 m high. A rough calculation using the equation for gravitational potential energy ($E_g = mg\Delta h$), which you learned in previous science courses, would yield a value of about 10 million joules of energy. Is there a way to avoid using so much energy?

Elevators are not usually simply suspended from cables. Instead, the supporting cable passes up over a pulley and then back down to a heavy, movable **counterweight**, as shown in Figure 1.11. Gravitational forces acting *downward* on the counterweight create tension in the cable. The cable then exerts an *upward* force on the elevator cage. Most of the weight of the elevator and passengers is balanced by the counterweight. Only relatively small additional forces from the elevator motors are needed to raise and lower the elevator and its counterweight. Although the elevator and counterweight move in different directions, they are connected by a cable, so they accelerate at the same rate.

Elevators are only one of many examples of machines that have large masses connected by a cable that runs over a pulley. In fact, in 1784, mathematician George Atwood (1745–1807) built a machine similar to the simplified illustration in Figure 1.12. He used his machine to test and demonstrate the laws of uniformly accelerated motion and to determine the value of g, the acceleration due to gravity. The acceleration of Atwood's machine depended on g, but was small enough to measure accurately. In the following investigation, you will use an Atwood machine to measure g.

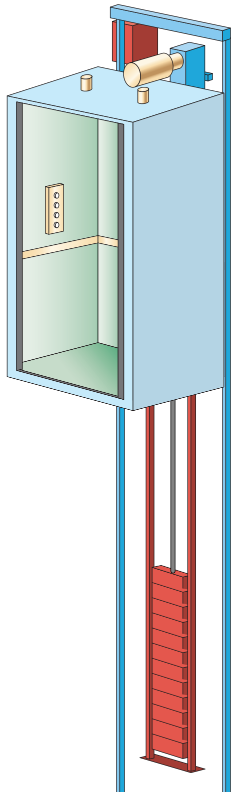

Figure 1.11 Most elevators are connected by a cable to a counterweight that moves in the opposite direction to the elevator. A typical counterweight has a mass that is the same as the mass of the empty elevator plus about half the mass of a full load of passengers.

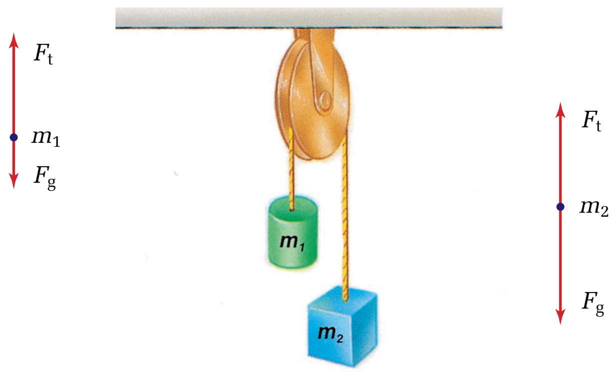

Figure 1.12 An Atwood machine uses a counterweight to reduce acceleration due to gravity.